MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

SUPPLEMENTARY EXAMINATION

TRIMESTER 1, 2015/2016

EEL1176 – FIELD THEORY

(All sections / Groups)

17 NOV 2015
2.30 PM – 4.30 PM
(2 HOURS)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 8 pages with 4 questions only.

2. Answer all **FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1

a) Two uniform vector fields are given by $\vec{P} = -5\hat{r} + 10\hat{\phi} + 3\hat{z}$ and $\vec{Q} = \hat{r} + 2\hat{\phi} - 6\hat{z}$, determine:

i) $\left(\vec{P} \times \vec{Q}\right)$. [2 marks]

ii) $\hat{z} \bullet \left(\vec{P} \times \vec{Q}\right)$. [2 marks]

iii) θ_{PQ}. [2 marks]

b) Consider the region defined by $2 < r < 5$, $\frac{\pi}{3} < \phi < \pi$, $3 < z < 4.5$. Find:

i) The enclosed volume. [3 marks]

ii) The total area of the enclosing surface. [3 marks]

iii) The length of the straight lines lies within the volume. [3 marks]

c) Refer to Figure Q1, verify Stoke's theorem for the vector field $\vec{A} = r\cos\phi\,\hat{r} + z\sin\phi\,\hat{z}$ by evaluating around L of the wedge defined by $0 \le r \le 2$, $0 \le \phi \le 60°$, z=0.

i) $\oint A \bullet dL$ over the contour [5 marks]

ii) $\int (\nabla \times A) \bullet dS$ [5 marks]

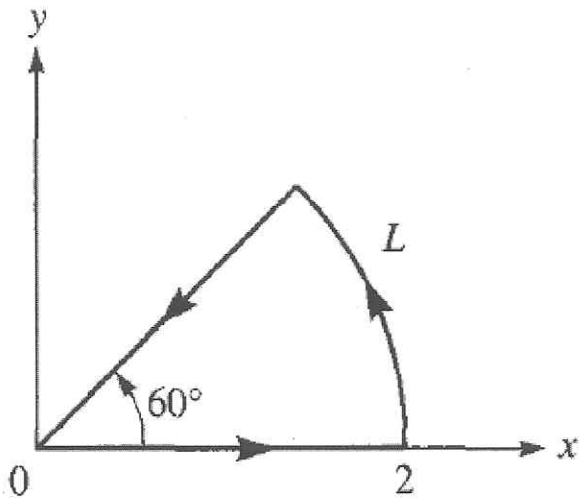

Figure Q1

Continued ...

Question 2

a) i) Explain the term differential electric potential (or differential voltage), dV. [2 marks]

ii) Give an expression of the electric potential due to N discrete point charges, q_1, q_2, …, q_N having position vectors, $\vec{R}_1$, $\vec{R}_2$, …, $\vec{R}_N$. [2 marks]

iii) Find the potential difference between two points due to a point charge q at the origin. [2 marks]

b) Calculate the work required to move a 5 C charge from point P_1 (1, 2, 3) to point P_2 (3, 1, 4) in the presence of an electric field $\vec{E} = \hat{x} + 2\hat{y} + \hat{z}$ V/m. The coordinates are given in unit of meter. [7 marks]

c) i) Describe the criteria to choose suitable Gaussian surfaces for the calculation of electric charges. [3 marks]

ii) A flat sheet is in the shape of a rectangle with sides of length 2 m and 4 m. The sheet is immersed in a uniform electric field of magnitude 20 V/m that is directed at 20^0 from the plane of the sheet. Find the magnitude of the electric flux through the sheet. [4 marks]

d) A parallel-plate capacitor of area A and separation d is charged to a voltage V. The permittivity of the dielectric is ε. If the fringing of the electric field at the edges is neglected, find the stored electrostatic energy in terms of capacitance and voltage. [5 marks]

Continued …

Question 3

a) The Region $0 \leq z \leq 2m$ is occupied by an infinite slab of permeable material ($\mu_r = 3$). If $\vec{B} = 8yx - 4x\hat{y}\ mWb/m^2$ within the slab, determine:

i) Free current density, $\vec{J}$. [4 marks]

ii) Bound volume current density, $\vec{J}_b$. [3 marks]

iii) Magnetization, $\vec{M}$. [4 marks]

iv) Bound surface current density, $\vec{K}_b$ on $z = 0$. [4 marks]

b) A square conducting loop of side 2a lies in the $z = 0$ plane and carries a current I in the counter-clockwise direction. Show that at the centre of the loop [10 marks]

$$\vec{H} = \frac{I\sqrt{2}}{\pi a}\hat{a}_z .$$

Continued ...

Question 4

a) Explain each of the following magnetic properties.
   i) Diamagnetism [5 marks]
   ii) Paramagnetism [5 marks]
   iii) Ferromegnetism [5 marks]

b) An air core toroid has a circular cross section of radius 4 mm. Calculate the inductance if there are 2500 turns and the mean radius of the toroid is 20 mm. [4 marks]

c) To simplify magnetic circuit analysis, magnetic circuit paramaters are analogy to an equivalent electric circuits. Complete Table Q4 with names and symbols.
[Redraw Table Q4 in your answer booklet]

Table Q4

Electric Circuits	Magnetic Circuits
Electromotive force, emf	
Conductivity, σ	
Electric current, I	

[6 marks]

End of Paper

Appendix A: Physical Constants, Vector and Coordinate Transformations

Elementary charge	e	1.60×10^{-19} C
Permittivity of free space	ε_0	8.85×10^{-12} F/m
Permeability of free space	μ_0	1.26×10^{-6} H/m $= 4\pi \times 10^{-7}$ H/m

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}\, r\, d\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}Rd\theta + \hat{\phi}R\sin\theta d\phi$
Differential surface areas	$d\vec{s}_x = \hat{x}\, dydz$ $d\vec{s}_y = \hat{y}\, dxdz$ $d\vec{s}_z = \hat{z}\, dxdy$	$d\vec{s}_r = \hat{r}\, rd\phi\, dz$ $d\vec{s}_\phi = \hat{\phi}\, dr\, dz$ $d\vec{s}_z = \hat{z}\, rdr\, d\phi$	$d\vec{s}_R = \hat{R}\, R^2 \sin\theta\, d\theta\, d\phi$ $d\vec{s}_\theta = \hat{\theta}\, R\sin\theta\, dR\, d\phi$ $d\vec{s}_\phi = \hat{\phi}\, R\, dR\, d\theta$
Differential volume	$(dx)(dy)(dz)$	$(dr)(r\, d\phi)(dz)$	$(dR)(R\, d\theta)(R\sin\theta\, d\phi)$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian & Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Spherical & Cartesian	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{x} = \hat{R}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{R}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical & Spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r}\sin\theta + \hat{z}\cos\theta$ $\hat{\theta} = \hat{r}\cos\theta - \hat{z}\sin\theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{r} = \hat{R}\sin\theta + \hat{\theta}\cos\theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Appendix B: Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi}r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Appendix C: Table of Integrals

$$\int \sin^2\theta d\theta = \frac{\theta}{2} - \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \cos^2\theta d\theta = \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \sin\theta\cos^2\theta d\theta = -\frac{1}{3}\cos^3\theta + \text{constant}$$

$$\int \cos\theta\sin^4\theta d\theta = \frac{1}{5}\sin^5\theta + \text{constant}$$

$$\int \sin 2\theta d\theta = -\frac{1}{2}\cos 2\theta + \text{constant}$$

$$\int x\sqrt{a^2 - x^2}\, dx = -\frac{1}{3}\left(a^2 - x^2\right)^{3/2} + \text{constant}$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + \text{constant}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} + \text{constant}$$

$$\int \frac{dz}{\left(r^2 + z^2\right)^{3/2}} = \frac{z}{r^2\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{rdr}{\left(r^2 + z^2\right)^{3/2}} = \frac{-1}{\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + \text{constant}$$

End of Appendix